Optimal Design of Wireless Mesh Networks

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Abstract. Wireless Mesh Networks (WMNs) are cost-effective and provide an appealing answer to connectivity issues of ubiquitous computing. Unfortunately, wireless networks are known for strong waste of capacity when their size increases. Thus, a key challenge for network operators is to provide guaranteed quality of service. Maximizing network capacity requires to optimize jointly the gateways placement, the routing and the link scheduling taking interferences into account. We present MILP models for computing an optimal 802.11a or 802.16 WMN design providing max-min bandwidth guarantee.

1 Introduction

There is an increasing interest in using Wireless Mesh Networks (WMNs) as next-generation broadband and ubiquitous access network [1]. In comparison to cellular, wireless single-hop, or wired networks [2], WMNs are indeed a scalable and cost-effective solution to collect information from mobile clients and send it to the Internet over a multi-hop wireless backhaul infrastructure.

A WMN is a fixed infrastructure of wireless routers, collecting and forwarding the traffic of mesh clients. This backhaul network interacts with other networks through special routers called gateways (Fig. 1). In WMNs, a high volume of traffic is expected to be efficiently delivered on the bandwidth-limited wireless channels, and a large number of users have to be fairly served [3]. Providing an end-to-end throughput guarantee is extremely valuable to network operators involved in WMN design and provisioning.

The study of wireless networks performance has motivated many research works. WMNs deployment in operational situations such as urban areas requires quality of service (QoS) criteria that are challenging to guaranty. Indeed, recent works have pointed out fundamental issues with capacity and scalability. Under specific routing, radio interference models and probabilistic traffic assumptions, it has been shown that random network performances degrade with a factor at least $\mathcal{O}(1/\sqrt{n})$ when the number of nodes, n, grows [4–6]. Generic capacity evaluation frameworks have been proposed using linear programming in order to get network behaviour estimations and stochastic analysis confirmations [7, 8]. Consequently, link scheduling protocols have been developed trying to cope with interferences among simultaneous transmissions [9].

The impact of interferences on routing has also been investigated [8, 10]. In order to deal with interference, it is important to know what are the sets of transmissions that can be active at the same time. These sets are usually called *rounds*. An algorithm enumerating a tractably large subset of simultaneous transmission rounds has been developed in order to compute an approximated solution for maximum throughput using

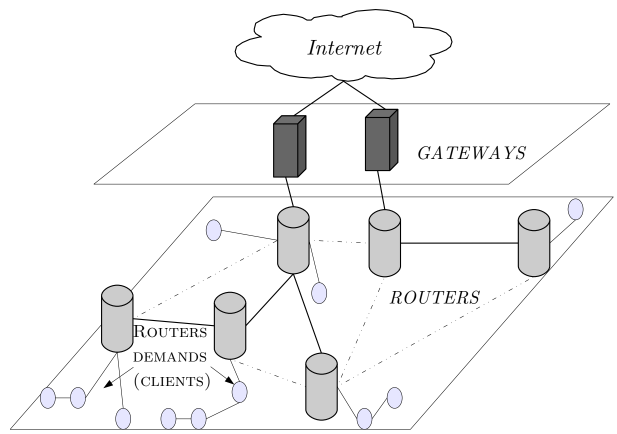


Fig. 1. An exemple of an Internet providing Wireless Mesh Network

linear programming [2]. For a multi-radio multi-channel network problem, a column generation approach has been developed for minimizing the time of the round scheduling [11]. Another approach consists in solving a multi-commodity flow problem by a primal-dual method in order to find upper bounds for the achievable throughput [12].

The joint routing and scheduling optimization in WMNs is a more recent topic motivated by the efficiency of cross layer approaches. It consists in computing jointly the routers-gateways routes of the packets, and the transmissions schedule in order to achieve the maximum transport capacity. This problem is closely related to the optimal assignment of interference-free broadcasting schedules in multi-hop packet radio networks, which has been shown NP-complete [13]. Another related problem, in sligthly different settings, is the *minimum time gathering* (MTG) [14] where each node has to send one unit of data to a central node within a global minimum gathering time. A 4-approximation algorithm has been developed for this problem [15].

The gateway placement has also a major impact on WMN QoS [16]. This problem is closely related to *facility location problems*, which have been extensively studied [17–20]. Nevertheless, only recent works include wireless interferences and their impact on the placement [16, 21]. In the similar *p-center* and *p-median* problems [22, 23], the number of facilities is given and the placement is optimized in order to minimize the distance to their associated clients. Again, multi-hop and interference concepts are not addressed in the litterature.

In this work, we address joint gateways placement, routing and scheduling problems. We focus on models computing optimal solutions for WMN design and provisioning. Throughput maximization does not model a fair bandwidth allocation. It may force some links to receive very low bandwidth or even to starve in order to reduce interferences [3]. To cope with QoS criteria interesting network operators, we investigate max-min throughput optimization which guaranty that each source node is allocated a lower bounded amount of bandwidth.

The remaining of this paper is organized as follows. Next section describes assumptions made about interferences and time division in our model. Section 3 presents the generic linear programming formulation for the design of WMNs, before presenting the different problems approached. Simulations are finally described in section 4.

2 Model and Assumptions

In the following, we assume that each wireless node transmits on a common isotropic channel with the same power and omnidirectional antennas. Communications between nodes create interferences such that no two interfering links can be activated simultaneously. The network is synchronous with slotted Time Division Multiplexing (TDM). Such a MAC model can be considered as a simplified version of 802.16 WiMax, or as a very optimistic approximation of 802.11 WiFi.

Moreover, all routers have infinite memory, meaning that we do not consider saturating networks where nodes are bottlenecks.

2.1 Interference model

A key issue in wireless networks optimization is to cope with interferences produced between nodes. Many interference models have been introduced, approximating the physical reality. Some are taking into account MAC layer protocols such as CSMA/CA with RTS-CTS. They are called binary models, meaning that two nodes can either always or never transmit simultaneously unless they communicate together [24].

Some other models try to fit a physical layer reality of Signal-and-Interference-to-Noise-Ratio (SINR) [4]. In this case, the success of receiving a transmission depends on the strength of the signal compared to the level of interference caused by simultaneous transmitting nodes added to the ambient noise.

However, most of these interference models are part of simple MAC layer. In the scope of this article, only one assumption is made: the interference model has to be described as set of incompatibilities, e.g. a conflict graph.

More formally, we assume that the WMN topology is given as a set of nodes V and a set of wireless links E. The interference model has to be expressed as $\mathcal{I} = \cup_{(u,v)\in E}\mathcal{I}(u,v)$, where $\mathcal{I}(u,v)$ is the set of links $e \in E$ that cannot be activated simultaneously with (u,v). With this generic formulation one can model a known interference scheme, the symmetric distance-2 interference model inspired by CSMA/CA, which we use for simulations in section 4.

We assume that all communications involve bidirectional messages exchanges, at least for acknowledgments, inducing a symmetric interference pattern. In this binary model, when a router transmits, all its neighbors keep silent. For sake of symmetric transmissions, the same happens with the receiver, inducing incompatibility patterns with the 2-hop neighborhood of an activated link :

$$\mathcal{I}(u,v) = \{(x,y) \in E \setminus \{(u,v)\} \mid \{x,y\} \cap (\Gamma(u) \cup \Gamma(v)) \neq \emptyset\},$$

where $\Gamma(u)$ represents the 1-hop neighboring nodes of u. In Figure 2, $\mathcal{I}(u,v)$ contains all links having an extremity in $\Gamma(u) = \{2\}$ or $\Gamma(v) = \{5\}$.

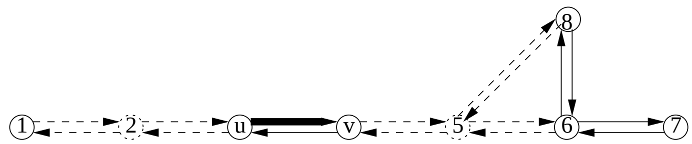


Fig. 2. Distance-2 Interference Model

2.2 Time multiplexing

We suppose a synchronous WMN with time decomposed into fixed slots. In each slot, a set of pairwise non-interfering links is activated. Such a set is called a *round* and corresponds to a stable set of the conflict graph.

For a flow of packets to be transmited from a router to a gateway, all the links along the (router,gateway)-path must be activated successively. We consider two kinds of transmissions : the permanent and the burst transmission. In the permanent transmission a flow of packets is permanently sent by the routers at a constant rate. On the contrary, a burst transmission happens at any moment when a router sends some units of flow (i.e. packets). A transmission is achieved when the last unit reaches its destination. Several burst transmissions may occur simultaneously from different routers.

In both cases, the network is periodic, each period consisting in a sequence of round activations. In the permanent transmission model, a relevant capacity measure is the mean or minimum flow, i.e. throughput. In the burst transmission model, the measure is the number of units of flow (packets) that can reach its destination in one time period. The order in which rounds are activated is thus of high importance, since it minimizes the delay between consecutive hops, while it has no impact on permanent transmissions.

3 Problems Definition

In this part, we design linear programming formulations that combine gateways placement, routing, and scheduling for WMNs. We first describe the problem constraints before presenting different objectives depending on the goal we want to achieve.

3.1 Modeling with ILP

A WMN can be described by a set V of wireless routers, a set E of wireless links, and a set $\mathcal{I}$ of interference sets, as defined in the previous section. Every link $e \in E$ has a capacity c_e, and every router has a traffic demand d_v computed as an amount of flow to send to the gateways through multi-hop paths. The considered time period is divided into slots $[1, T]$ during which a set of pairwise non interfering links carries flow. Let us define the following variables for the network design :

- $f_e^r \geqslant 0$ represents flow from router r that passes through link e,
- $y_{r,i} \geqslant 0$ represents traffic flow sent by router r that reaches gateway i,
- $a_e^t \in \{0, 1\}$ represents activation of link e during time slot t,
- $s_i \in \{0, 1\}$ represents selection of router i to be a gateway.

Then, the computed link-scheduling will permit flow transmissions from routers to gateways taking into account wireless interferences.

$$a_e^t + a_{e'}^t \leqslant 1,\ \forall e \in E, e' \in \mathcal{I}(e), t \leqslant T \tag{1}$$

$$\sum_{v \in V} f_e^v \leqslant c_e \cdot \sum_{t \leqslant T} a_e^t,\ \forall e \in E \tag{2}$$

$$\sum_{e=(u,v)\in E} f_e^r + \mathbb{I}_{\{v=r\}} d_v = \sum_{e=(v,w)\in E} f_e^r + y_{r,v},\ \forall r, v \in V \tag{3}$$

$$\sum_{i \in V} y_{r,i} = d_r,\ \forall r \in V \tag{4}$$

$$y_{r,i} \leqslant s_i \cdot C,\ \forall i \in V, r \in V \tag{5}$$

(1) are the interference constraints: if link e is active at time t, no link in interference set $\mathcal{I}(e)$ can be activated simultaneously. (2) are the link capacity constraints: the total flow on a link cannot exceed its global capacity on the time period, which is the number of slots the link is activated times its nominal capacity c_e. (3) are the flow conservation constraints where r is the source of the flow f and v is the current node that forwards the received flow following Kirchhoff laws. Two special cases happen : when $v = r$, then the router sends its demand on its ougoing edges. If v is a gateway, it can receive an amount of the flow sent by r, i.e. $y_{r,v}$. (4) claims that all demands must be received by the gateways. And (5) claims that the absorbed flow by a router is null if this one is not selected to be a gateway, i.e. $s_i = 0$ (C is a constant greater than the total demand).

3.2 WMN optimization

The previous ILP describes the polytope containing all WMN configurations. By setting some variables and specifying an objective function, a particular WMN optimization is defined. In the following, we present three WMN design problems.

Optimal Gateways Placement Problem (GPP): When traffic demands are known for every router, the gateways placement problem seeks to deploy gateways in the WMN such that all demands are satisfied. The objective (6) is then to minimize the number of selected gateways in the network.

A subset of nodes can be defined as candidates to become gateways, and we only try to select a minimum number of them.

$$\textbf{Objective :}\ \min \sum_{i \in V} s_i \tag{6}$$

Fair Gateways Placement Problem (FGPP): If the number of gateways we want to deploy is known, the objective is to place this number of gateways in order to maximize the throughput within a simplified max-min fairness model, where the minimum throughput assigned to a router is maximum among all feasible routing and schedules (obj.(7)).

As in GPP, we can have a subset of candidates routers and select a fixed number n of them for the placement (eq.(8)).

$$\textbf{Objective:} \quad \max(\min_{v \in V}(d_v)) \tag{7}$$

$$\textit{such that:} \quad \text{eq.(1)-(5) and } \sum_{i \in V} s_i = n \tag{8}$$

Fair Routing and Scheduling Problem (FRSP): In this case, an achieved gateways placement is known. All s_i are constants and equal 1 for the set of gateways, 0 otherwise. The fair routing and scheduling problem is then finding the maximum throughput that can be guaranteed to each router of the WMN, leading to objective (7) as in FGPP.

3.3 Burst Transmission

When considering burst transmissions, the order of the activation of the links during the scheduling matters. When a router is ready to send packets, there is an initial delay before the first link of the path being activated. Thus, there are time slots when no transmission happens. Oppositely for permanent transmissions, all links have flow to carry at any time slot. We observe that this initial delay has a huge impact in short burst transmissions, while it has obviously no impact on the permanent case.

To reduce the initial delay in the burst case we consider *ordered* schedulings with the following property: if a path is used to send packets from a router r to a gateway g, then links of the path must be activated in an increasing order from r to g. Thus, a flow passes through a link only if it has already passed through the previous link in the path.

Figure 3 shows a very simple example to be clear. In this example, time period is $[1, T]$ with $T = 5$ in both cases. We can observe two different link schedules for paths going from the routers to gateway node 5. Ordered (on the left) and not ordered scheduling (on the right) are presented. The initial delays are underlined in each case.

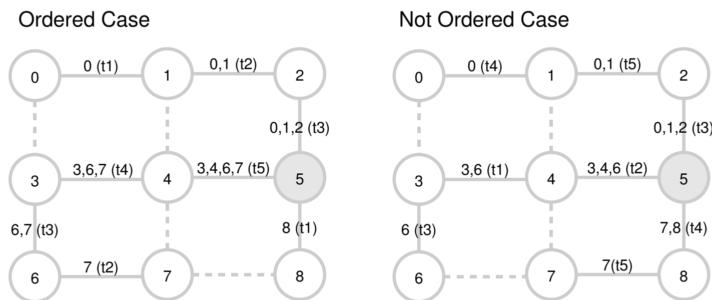
The worst case to a router node in the ordered sequence happens when the necessity to send a burst arrives one slot after the first slot of the path schedule. In the best case, the necessity to send arrives at the first slot of the path schedule.

As seen in the ordered scheduling, the best case occurs when router 1 starts sending at slot $t = 2$. After that, only two slots are needed to complete the transmission. In the worst case, router 1 is ready at $t = 3$ but it should wait the time slot $t = 2 + T$ to start the transmission. In this case, the router 1 got an initial delay of 4 slots, $3, 4, 5, 1 + T$.

We add the constraint (9) in the model to tackle burst transmissions, which activates the links of a path in an increasing order. The variable f_e^v in the model is then decomposed into the sum over the time period of the flows $x_e^{v,t}$ from router node v over link e during time slot t.

$$\sum_{\substack{t1 \in T \\ t1 < t}} \sum_{\substack{u \in V \\ e=(u,v) \in E}} x_e^{r,t1} \geqslant \sum_{\substack{t2 \in T \\ t2 \leqslant t}} \sum_{\substack{w \in V \\ e=(v,w) \in E}} x_e^{r,t2}, \forall r, v \in V, t \leqslant T \tag{9}$$

Note that, for the burst trasmission model we are considering just packets gathering from the routers to the gateway. It is just to ilustrate that the order of the links makes difference in not permanent transmissions.



Node	Min. time	Max. Time
0	slot **1,2,3**=3	slot 2,3,4,5,**1,2,3**=7
1	slot **2,3**=2	slot 3,4,5,1,**2,3**=6
2	slot **3**=1	slot 4,5,1,2,**3**=5
3	slot **4,5**=2	slot 5,1,2,3,**4,5**=6
4	slot **5**=1	slot 1,2,3,4,**5**=5
6	slot **3,4,5**=3	slot 4,5,1,2,**3,4,5**=7
7	slot **2,3,4,5** =4	slot 3,4,5,1,**2,3,4,5**=8
8	slot **1**=1	slot 2,3,4,5,**1**=5

Node	Min. time	Max. Time
0	slot **4**,5,1,**2,3**=5	slot 5,1,2,3,**4,5**,1,2,**3**=9
1	slot **5**,1,2,**3**=4	slot 1,2,3,4,**5**,1,2,**3**=8
2	slot **3**=1	slot 4,5,1,2,**3**=5
3	slot **1,2**=2	slot 2,3,4,5,**1,2**=6
4	slot **2**=1	slot 3,4,5,1,**2**=5
6	slot **3**,4,5,**1,2**=5	slot 4,5,1,2,**3**,4,5,**1,2**=9
7	slot **5**,1,2,3,**4**=5	slot 1,2,3,4,**5**,1,2,3,**4**=9
8	slot **4**=1	slot 5,1,2,3,**4**=5

Fig. 3. Initial delay penalty.

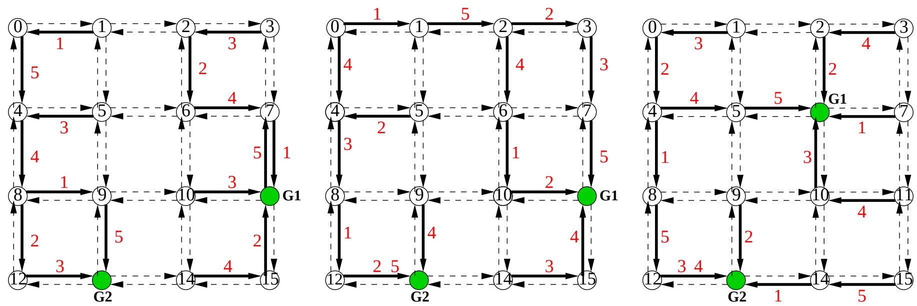
4 Results

The three models presented in section 3.2 were implemented with the MASCOPT[1] library, and solved using ILOG CPLEX. We validate our models through simulations on grid and random mesh topologies using the distance-2 binary interference model described in section 2.

Even for small networks and considering small time periods, the program generates MILPs with thousands of constraints and variables. Therefore, large instances cannot be solved to optimality and only approximate solutions can be obtained. The approximation factor is denoted ”Gap” in Table 1 which summarizes our simulation results.

Results highlight the fact that even in small networks, gateways placement has a major impact on the network throughput. If the GPP objective is considered, we will get a placement with minimum number of gateways to attend a specific demand. Then FRSP computes the maximum demand each router can send according to this placement. For instance, we need only two gateways on a 4 by 4 grid to attend a given demand of 5, with an initial link capacity of 20 (Figure 4(a)). But with the same placement, it is possible to increase the throughput with FRSP objective. Given the placement found by GPP as input of FRSP, the max-min throughput can therefore be better. Figure 4(b) shows that the placement of two gateways at nodes 11 and 13 calculated for a fixed demand of 5 for each router, can actually satisfy a demand of 6 with the same time period.

[1] Mascotte Optimization *http://www-sop.inria.fr/mascotte/mascopt*



(a) GPP (throughput=5) (b) FRSP (throughput=6) (c) FGPP (throughput=8)

Fig. 4. Grid 4x4 results with T=5. Numbers on links represent activated time slots.

Finaly, with the FGPP model, we can search for a new placement that gives the maximum fair throughput, given a fixed number of gateways. If we use only the number of gateways found in GPP (e.g. 2), then FGPP finds a better throughput of 8 units by time period as shown in Figure 4(c) with a new placement of two gateways. Demands given in GPP might not be the maximum one, which is actually computed by FRSP for the same placement. But maybe the placement found is not optimal for the same number of gateways, and a better one should also increase the network throughput. Thus, FGPP will find a new placement that is optimal for the maximum throughput.

We obtain another results with the 4 by 4 grid and $T = 4$. GPP gives two gateways to attend a demand of 5, and FRSP shows that it is the maximum demand for this placement. But FGPP finds another placement of two gateways where it is possible reach a throughput of 6 units by time period.

FRSP is more tractable since there are less binary variables in its formulation. Therefore more results, with one gateway, have been found and are described in Table 1. The topology name *grid3Vg4* represents a 3 by 3 grid with the gateway located in position 4 (the center). We can see, for instance, that the reached throughput on this topology is 25 units with a time period of 5 time slots. If we put the gateway at position 5, a throughput of 33 is achievable. But when we increase the time period to 6 we get a better throughput with the gateway located in 4 than the gateway in 5, respectively 50 and 40 units by time period, since links incident to node 4 can be activated enough to route more demand. We can observe that the model dealing with the burst traffic gives a lower bound to the FRSP with permanent transmission because the ordering leads to a more restricted scheduling.

5 Conclusion

In this paper, we address gateways placement, fair routing and scheduling problems for WMNs where our major concern is optimal placement, fairness and throughput guarantee. We introduced a generic polytope describing all valid WMN configurations and three optimal problems based upon it. A Mixed Integer Linear Programming (MILP) formulation has been developed with different objectives that either gives the maximum

Table 1. Results

		Permanent Transmission			Burst Transmission		
Topology	time slots	run time(s)	Gap(%)	throughput	run time(s)	Gap(%)	throughput
line7Vg0	10	4.39	0	60	4.75	0	60
grid3Vg4	5	286.31	0	25	26	0	25
grid3Vg5	5	67.98	0	33	4.87	0	25
grid3Vg2	6	130.67	0	40	11.03	0	33
grid3Vg4	6	98.22	0	50	14.9	0	50
grid3Vg5	6	775.31	0	40	30.5	0	37
grid5Vg12	10	1705.20	20	29	1630.47	20	29
grid5Vg4	10	1068.41	20	25	1049.36	20	25
grid5Vg12	20	16362.83	20	62	16411.11	20	62
grid7Vg24	15	40998.59	20	21	43539.55	20	21
mesh6Vg1	5	1.50	0	50	0.21	0	50
mesh8Vg4	5	110.57	0	66	11.75	0	66
mesh11Vg1	10	9444.22	3	33	29827.77	20	40

throughput that can be guaranteed for every routers in the network, or the minimum number of gateways we need to deploy to satisfy the traffic demand. GPP gives a minimum gateways placement with a fixed routers demand. Given a placement, FRSP finds the maximum fair throughput. And FGPP combines the previous problems, providing the best placement for a fixed number of gateways, to reach the maximum throughput.

Time multiplexing is a good answer to model radio interferences but makes the number of binary variables increase significantly. Using a MILP model allows to find optimal or near-optimal solutions for small and medium sized instances. The difficult task is to prove optimality of the solution found. In order to tackle larger networks, more sophisticated approaches are under investigation. Call scheduling for permanent transmission is related to arc-list coloring with specific constraints. A graph theoretic study should give results or bounds decreasing the complexity of the problem. Moreover, a linear formulation based on more complex combinatorial objects is being designed in order to develop efficient column or line generation algorithms that are promising.

A recursive approach of the problem seems to be relevant because of the network clustered-pattern: a gateway concentrates traffic from a cluster which might be connected and even convex.

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